A New Type of Hilbert Modules

Harald Upmeier
University of Marburg, Germany

Classically, Hilbert modules are considered for (multivariable) holomorphic functions. In this talk we introduce a new type of Hilbert modules which do not consist of holomorphic functions but instead involve Dolbeault cohomology classes of differential forms of higher degree. We give examples from representation theory (discrete series of semi-simple Lie groups) which show that the holomorphic case gives only a small fraction of the theory, whereas the cohomology approach yields a much more complete picture. We also show that many fundamental problems studied in the holomorphic case can also be addressed for this new type of Hilbert modules. An interesting feature of the new approach is that it is already non-trivial in the strongly pseudo-convex case of the unit ball. This will be explained in detail in the lecture.

Doubly commuting invariant subspaces of $L^2(\mathbb{T}^n)$

Shuaibing Luo(罗率兵)
Hunan University, China

In this talk, we discuss the invariant subspace M for the tuple of multiplication opeators $M_z = (M_{z_1}, \cdots, M_{z_n})$ on $L^2(\mathbb{T}^n)$ under the condition that the restricted tuple $M_z \mid M = (M_{z_1}|M, \cdots, M_{z_n}|M)$ is doubly commuting. We also formulate our invariant subspace theorem using intersecting families of subsets of $\{1, 2, \cdots, n\}$. As applications of our result, we obtain invariant subspace theorems for tuple of shift related operators including Beurling type theorems on the Hardy space of the polydisk.

Commuting tuple of multiplication operators homogeneous under the unitary group

Soumitra Ghara
IIT Kharagpur, India

Let $\mathcal{U}(d)$ be the group of $d \times d$ unitary matrices. In this talk, we will discuss conditions to ensure that a $\mathcal{U}(d)$-homogeneous d-tuple $\boldsymbol{T}$ is unitarily equivalent to the operator of multiplication by the coordinate functions on some reproducing kernel Hilbert space $\mathcal{H}_K(\mathbb{B}_d, \mathbb{C}^n) \subseteq \mathrm{Hol}(\mathbb{B}_d, \mathbb{C}^n), n = \dim \cap_{j=1}^d \ker T_j^*$. We will describe this class of $\mathcal{U}(d)$-homogeneous operators, equivalently, non-negative kernels K quasi-invariant under the action of $\mathcal{U}(d)$. We will also present a classification of quasi-invariant kernels K transforming under $\mathcal{U}(d)$ with two specific choice of multipliers. An explicit criterion for boundedness, reducibility and mutual unitary equivalence among these operators will also be discussed. This is a joint work with S. Kumar, G. Misra and P. Pramanick.

Toeplitz operators and α-Berezin transforms on the Bergman space

Fugang Yan(晏福刚)
Chongqing University, China

The characterization of bounded Toeplitz operators with integrable symbols in the Bergman space is an open problem even in the simplest case of the unweighted Bergman space of unit disc. In this talk, we will introduce the recent progresses related to this problem. In particular, we study this problem via α-Berezin transforms. We show that if a Toeplitz operator T_f with $f \in L^2(\mathbb{D})$ is bounded, then α-Berezin transform of f is bounded for all $\alpha > -1/2$. We also show that for $-1 < \alpha < 1$, the boundedness for the α-Berezin transform of f cannot imply the boundedness of the Toeplitz operator T_f. This talk is based on a joint work with professor Dechao Zheng.

Fractional derivatives on Bloch and Bergman spaces

Siyu Wang(王司宇)
Fudan University, China

In this talk, we will show that the classical Bloch space and the Bergman space A^p_η, induced by a radial doubling weight η, can be characterized by using the fractional derivative

$$f \mapsto R_{\nu,\omega}(f)(z) = \sum_{k=0}^{\infty} \frac{\nu_{2k+1}}{\omega_{2k+1}} \widehat{f}(k) z^k, \quad z \in \mathbb{D},$$

induced by two radial weights admitting certain doubling conditions. Here

$$\omega_{2k+1} = \int_0^1 r^{2k+1} \omega(r) dr$$

are the odd moments of ω, and $\widehat{f}(k)$ stands for the Maclaurin coefficient of the analytic function f in the unit disc $\mathbb{D}$.

The arguments employed here rely on the natural integral representation

$$R_{\nu,\omega}(f)(z) = \int_{\mathbb{D}} f(\zeta) \overline{B^\omega_z(\zeta)} \nu(\zeta) dA(\zeta), \quad z \in \mathbb{D}, \quad f \in L^1_\nu$$

via the Bergman reproducing kernels B^ω_z of the Hilbert space A^2_ω. On the path to our main result, we also study the boundedness of fractional derivative $R_{\nu,\omega}$ as an integral operator from weighted L^∞ to weighted Bloch spaces. This talk is based on a joint work with A. Perälä and J. Rättyä.

Critical points, finite Blaschke products and Riemann surface: on multiplication operators over the Bergman space

Hansong Huang(黄寒松)
East China University of Science and Technology, China

In this talk, we first review recent advances in characterizing the reducing subspaces of multiplication operators induced by several classes of inner functions on the Bergman space $L_a^2(\mathbb{D})$ over the unit disk. Specifically, for a finite Blaschke product B, it is known that the von Neumann algebra $\mathcal{V}^*(B) = \{M_B, M_B^*\}'$ is abelian, where M_B is the multiplication operator defined by B on the Bergman space $L_a^2(\mathbb{D})$. It was proven that the number of minimal projections in $\mathcal{V}^*(B)$ is equal to that of components of the Riemann surface $\mathcal{S}_B$contained in $\mathbb{D}^2$. However, determining this integer for a specific finite Blaschke product B is a challenging task. We take an approach by analyzing the analytic continuations and identifying its critical points. This approach reveals an interplay between function theory, operator theory, and complex geometry.

This is a joint work with Danni Guo, Shan Li, Shuaibing Luo.

Pairs of compatible isometries - model, spectrum and applications

Zbigniew Burdak
University of Agriculture in Krakow, Poland

Pair of isometries is called compatible if projections on powers of range spaces commute for any pair of powers. In the talk we recall the model, show an application to stochastic processes and calculate Taylor spectrum in some cases. The application describes evanescent part for weakly stationary stochastic process with a half-plane past. Taylor spectrum in the case of pairs given by diagrams (which is one of the types in the model) is a proper subset of a bi-disk and may be of any measure in $(0, \pi^2)$.

von Neumann's inequality on the polydisc

Michael Hartz
Saarland University, Germany

The Schur-Agler norm of a d-variable polynomial p is defined as

$$\|p\|_{SA} = \sup\{\|p(T_1, \dots, T_d)\| : T_1, \dots, T_d \text{ commuting contractions on Hilbert space}\}.$$

Classical results of von Neumann and Andô show that if $d = 1$ and $d = 2$, then the Schur-Agler norm equals the supremum norm on $\mathbb{D}^d$. If $d \geq 3$, this is no longer true, but it is still now known whether the Schur-Agler norm is equivalent to the supremum norm.

I will talk about new upper and lower bounds for the Schur-Agler norm of homogeneous polynomials in $d \geq 3$ variables. This partly joint work with Dexie Lin and Yi Wang.

On the classification of $\mathcal{J}$-holomorphic curves

Kui Ji(纪奎)
Hebei Normal University, China

Let $\mathcal{H}$ be a complex separable Hilbert space. For Ω an open connected subset of $\mathbb{C}$, we shall say that a map $f : \Omega \to \mathrm{Gr}(n, \mathcal{H})$ is a holomorphic curve, if there exist n holomorphic $\mathcal{H}$-valued functions $\gamma_1, \gamma_2, \cdots, \gamma_n$ on Ω such that

$$f(w) = \bigvee \{\gamma_1(w), \gamma_2(w), \cdots, \gamma_n(w)\}, w \in \Omega,$$

where $\mathrm{Gr}(n, \mathcal{H})$ denotes the Grassmann manifold, the set of all n -dimensional subspaces of $\mathcal{H}$. The homogeneous curves, weakly homogeous curves in Cowen-Douglas class are introduced A. Koranyi and G. Misra. The relevant researches on this type of curves has established the relationship between group representation theory, operator theory, Hilbert module and complex geometry.

In this talk, we will introduce a new and large class of holomorphic curves named as $\mathcal{J}$-holomorphic curves which including homogeneous curves, weakly homogeous curves. By using some geometric invariants, we also give a similarity classification theorem of this kind curves.

The Fourier decay of cascade measures

Zipeng Wang(王子鹏)
Chongqing University, China

I will report our solutions to the Mandelbrot problem/conjecture (1974) and the Kahane problem (1993), which concern the optimal Fourier decay of cascade measures originally motivated by the K41 turbulence model.

This report requires no prior knowledge of probability theory and is particularly wellsuited for experts and scholars with a background in functional analysis.

This is a series of joint works with Xinxin Chen, Yong Han, and Yanqi Qiu.

Carelson Embedding for doubling weighted Bergman-Orlicz spaces and its applications

Yongjiang Duan(段永江)
Jinan University, China

We provide geometric characterizations for the positive Borel measures μ such that the weighted Bergman-Orlicz space $A^{\Phi_1}_{\omega}$ is continuously embedded into $L^{\Phi_2}_{\mu}$, where Φ_1, Φ_2 are Young functions satisfying certain growth conditions, and the radial weight ω on the unit disk satisfies the doubling condition. As applications, we give the characterizations of the Toeplitz operators, Volterra operators and composition operators between the doubling weighted Bergman-Orlicz spaces. This is joint with Min Dong, Kunyu Guo and Siyu Wang.

Converse to Lempert's theorem and extension sets

Lukasz Kosinski
Jagiellonian University, Poland

We will recall Lempert's theorem, explain how it arises in Several Complex Variables, and describe its connections with Operator Theory. We will then discuss the extent to which it works and how a converse arises in the context of the Extension Property. The talk is based on joint work with J. E. McCarthy and J. Agler.

On weakly $\mathcal{U}(d)$-homogeneous Tuple of Operators

Surjit Kumar
IIT Madras, India

The unitary group $\mathcal{U}(d)$ acts naturally on any $_d$-tuple $\boldsymbol{T} = (T_1, \ldots, T_d)$ of commuting bounded linear operators on a complex separable Hilbert space $\mathcal{H}$. A commuting $_d$-tuple $T = (T_1, \ldots, T_d)$ is called weakly $\mathcal{U}(d)$-homogeneous if, for every $u \in \mathcal{U}(d)$, there exists a bounded invertible operator $\Gamma(u)$ on $\mathcal{H}$ such that

$$\Gamma(u)T_j = (u \cdot T)_j \Gamma(u), \quad j = 1, \ldots, d.$$

Every $\mathcal{U}(d)$-homogeneous tuple is clearly weakly $\mathcal{U}(d)$-homogeneous. Furthermore, any commuting $_d$-tuple that is similar to a $\mathcal{U}(d)$-homogeneous tuple must be weakly $\mathcal{U}(d)$ homogeneous. In this talk, we identify a few classes of weakly $\mathcal{U}(d)$-homogeneous operator tuples that are, in fact, similar to $\mathcal{U}(d)$-homogeneous tuples. One such class is the family of $_d$-variable weighted shifts. In this class, a spherical isometry is weakly $\mathcal{U}(d)$-homogeneous if and only if it is similar to the Szegő shift.

This is a joint work with Soumitra Ghara, and Shailesh Trivedi.

Commutants of composition operators on Fock space

Maofa Wang(王茂发)
Wuhan University, China

This talk will give some profound and comprehensive characterizations on commutants of composition operators on Fock space of several variables. Also we consider the asymptotic behaviour of discrete or continuous semigroups of such operators. This is joint work with Frédéric Bayart and Xingxing Yao.

Characteristic functions and invariant subspaces: old and new

Dan Timotin
Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

In the Sz.Nagy-Foias theory of contractions on a Hilbert spaces, an important point is the identification of a one-to-one correspondence between the invariant subspaces of the contraction and certain factorizations of its characteristic function. We will discuss this correspondence, including details on its history. Subsequently we will present some recent applications and extensions, in view also of possible generalizations.

Cubic matrix Szegő equation and Hankel operators

Pan Ma(马攀)
Central South University, China

In this talk, we will present some progress on the flow of cubic matrix Szegő equation on the unit circle $\mathbb{T}$ which is the evolution equation on $H^2\left(\mathbb{T}, \mathbb{C}^{m\times n}\right)$

$$i\partial_t U = \mathbb{P}\left(UU^*U\right), U = U(t,x) \in \mathbb{C}^{m\times n}, (t,x) \in \mathbb{R}\times\mathbb{T},$$

where $\mathbb{P}$ denotes the matrix Szegő projection from $L^2\left(\mathbb{T}, \mathbb{C}^{m\times n}\right)$ onto $H^2\left(\mathbb{T}, \mathbb{C}^{m\times n}\right)$.

Inner and characteristic functions in polydiscs

Deepak Kumar Pradhan
IIT Hyderabad, India

A $_d$-tuple of contractions $\boldsymbol{T} \in \mathcal{B}(\mathcal{H})^d$ is said to be a pure Szegö tuple if $\prod_{i=1}\left(I - T_iT_i^*\right) \geq 0$ and $T_i^{*n} \xrightarrow{SOT} 0$. Further, a pure Szegö tuple $\boldsymbol{T}$, is said to be of Beurling type if the model space of $\boldsymbol{T}$ is a Beurling quotient module in the Hardy space of polydisc. In this talk, we will define the operator-valued characteristic function $\Theta_{\boldsymbol{T}}(z)$ on the polydisc $\mathbb{D}^n$ for pure Szegö tuple $\boldsymbol{T}$ of Beurling type. We also show that $\Theta_{\boldsymbol{T}}(z)$ is a complete unitary invariant for a pure Szegö tuple $\boldsymbol{T}$ of Beurling type. This talk is based on a joint work with Prof. Jaydeb Sarkar and Dr. Ramlal Debnath.

Boyd indices and the complete story of the Berger-Coburn phenomenon

Jingbo Xia(夏经博)
University of Buffalo, USA

For a symmetrically normed ideal $\mathcal{C}_\Phi$, its Boyd indices p_Φ and q_Φ are wellunderstood scalar

invariants, and they always satisfy the inequality $1 \leq p_\Phi \leq q_\Phi \leq \infty$. We settle the issue of Berger-Coburn phenomenon on the Fock space completely for general symmetrically normed ideals $\mathcal{C}_\Phi$, where $\|\cdot\|_\Phi$ is not equivalent to the operator norm $\|\cdot\|$. We show that if the Boyd indices of $\mathcal{C}_\Phi$ satisfy the condition $1 < p_\Phi \leq q_\Phi < \infty$, then for $f \in L^\infty(\mathbf{C}^n)$, we have $H_f \in \mathcal{C}_\Phi$ if and only if $H_{\bar{f}} \in \mathcal{C}_\Phi$. We further show that if either $p_\Phi = 1$ or $q_\Phi = \infty$, then there is an $f \in L^\infty(\mathbf{C}^n)$ such that $H_f \in \mathcal{C}_\Phi$ while $H_{\bar{f}} \notin \mathcal{C}_\Phi$.

A joint spectral radius for tuples of operators

Orr Shalit
Technion - Israel Institute of Technology, Israel

The study of algebras of bounded noncommutative analytic functions on subvarieties of noncommutative unit balls has led us to associate a spectral radius function ρ_E with every finite-dimensional operator space E. Concretely, if $A = (A_1, \ldots, A_d)$ is a d-tuple of operators, then $\rho_E(A)$ is defined via a certain tensor power limit formula, which reduces to Gelfand's spectral radius formula when E is one-dimensional. When E is the row operator space, ρ_E coincides with the joint spectral radius studied by Bunce, Popescu and others.

In a recent paper with Eli Shamovich, we introduced the joint spectral radius ρ_E and proved that $\rho_E(A) < 1$ if and only if A is jointly similar to a tuple that lies in the noncommutative ball corresponding to E. For example, when E is the row operator space, $\rho_E(A) < 1$ means that A is jointly similar to a strict row contraction. In this talk, based on joint work with Eli Shamovich, I will explain why we were led to this notion, present some examples, and describe possible applications. Finally, I will discuss recent progress on the case of commuting tuples, where several alternative notions of joint spectrum coincide.

Helton-Howe trace formulae on quotient modules

Lijia Ding(丁立家)
Zhengzhou University, China

In this talk, we will discuss Helton-Howe trace formulae on Hilbert modules over Stein strongly pseudoconvex finite manifolds. For higher-dimensional smooth subvarieties that transversely intersect with the strongly pseudoconvex boundary, we establish an integral trace formula for the compression of Toeplitz operators on holomorphic Sobolev quotient modules, provided the restriction maps satisfy a q-essentially isometric condition for some q strictly less than the dimension of the subvariety. Our findings extend known results from the unit ball to the Stein strongly pseudoconvex manifolds and partially answer a problem recently proposed for the Helton-Howe trace formula on quotient modules. This talk is based on a joint work with Prof. Kunyu Guo and Prof. Kai Wang.

Extension of contractive projections

Xiangdi Fu(付祥迪)
Hangzhou Institute for Advanced Study, UCAS, China

Through the establishment of several extension theorems, we provide explicit expressions for all contractive projections and 1 -complemented subspaces in the Hardy space $H^p(\mathbb{T})$ for $1 \leq p < \infty, p \neq 2$. Our characterization leads to two corollaries: first, all nontrivial 1 -complemented subspaces of $H^p(\mathbb{T})$ are isometric to $H^p(\mathbb{T})$; second, all contractive projections on $H^p(\mathbb{T})$ are restrictions of contractive projections on $L^p(\mathbb{T})$ that leave $H^p(\mathbb{T})$ invariant. The first corollary provides examples of prime Banach spaces in the isometric sense, while the second answers a question posed by P. Wojtaszczyk in 2003. This talk is based on a joint work (arXiv:2412.14808) with Professor Kunyu Guo and Dr. Dilong Li.

Some Clark Theory on the Bidisk

Kelly Bickel
Bucknell University, USA

Classical Clark measures are singular measures on the unit circle defined via inner functions that are closely tied to important topics in operator theory and complex analysis (for example, model spaces, compressed shifts, and composition operators). In this talk, we'll consider an analogous definition for Clark measures associated with twovariable inner functions. In the rational case, we'll give exact formulas for these Clark measures and characterize when associated Clark embeddings are unitary. For certain classes of two-variable model spaces, we will also use this theory to obtain nice unitary

Some results on analytic function spaces by potentials

Guanlong Bao(鲍官龙)
Shantou University, China

We talk about some results on analytic function spaces in terms of Newtonian potentials and certain variants of the Green potential. These results are about the problems of determining Carleson measures for the Bloch space and characterizing zero sets for Dirichlet spaces with harmonic weights.

Dynamical Frames and Hyperinvariant Subspaces

Victor Bailey
University of Oklahoma, USA

The theory of dynamical frames arose from practical problems in dynamical sampling where the initial state of a vector needs to be recovered from the space-time samples of future states of the vector. This leads to the investigation of structured frames obtained from the orbits of evolution operators. One of the basic problems in dynamical frame theory is to determine the semigroup representations, which we will call central frame representations, whose frame generators are unique (up to equivalence). In this talk, we will address the general uniqueness problem by presenting a characterization of central frame representations for any semigroup in terms of the co-hyperinvariant subspaces of the left regular representation of the semigroup. This result is not only consistent with the known result of Han and Larson in 2000 for group representation frames, but also proves that the frame vectors for any system of the form $\{A_1^{n_1} \cdots A_k^{n_k} : n_j \geq 0\}$, where $A_1, \ldots, A_k \in B(H)$ commute, are equivalent. This is joint work with Deguang Han, Keri Kornelson, David Larson, and Rui Liu.

Subnormality of Brownian-type operators

Sameer Chavan
IIT Kanpur, India

In this talk, we discuss operators that are represented by upper triangular 2×2 block matrices whose entries satisfy some algebraic constraints. We call them Browniantype operators. These operators emerged from the study of Brownian isometries studied by Agler and Stankus via detailed analysis of the time shift operator of the modified Brownian motion process. We address the issue of subnormality of Brownian-type operators and relate it to a spectral inclusion of a pair of commuting normal operators.

This talk is based on a joint work with Z. J. Jabłoński, I. B. Jung and J. Stochel.

Measures on General Co-dimensional Surfaces in Innite Dimensions and Stokes Type Theorems

Jiayang Yu(余佳洋)
Sichuan University

In this talk, we construct explicitly measures on general co-dimensional surfaces in ℓ^2 which is, in some sense, the simplest infinite-dimensional space, closest to Euclidean spaces, but the construction of related surface measures is a longstanding unsolved problem. Our surface measures are naturally induced from the usual Gaussian measures on ℓ^2, based on which we introduce the notion of differential forms and establish the corresponding Stokes type theorems.

Constrained sphere packing and the interpolation problem for positive definite functions

James Eldred Pascoe
Drexel University, USA

Delsarte's method provides a framework for analysis of sphere packing problems in terms of positive definite functions. Constrained packing, where only a prespecified set of angles between points may occur, is, in turn, naturally governed by the interpolation theory for positive definite functions. In this talk, we discuss the general theory of positive definite functions on the sphere, Delsarte's method and its geometric kernel embedding interpretation, related interpolation problems, and other topics. Based on joining work with Sujit Sakharam Damase.

Essential normality of quotient modules vs Hilbert-Schmidtness of submodules in Hardy modules over the polydisc

Penghui Wang(王鹏辉)
Shandong University, China

In this talk, I will introduce the recent development on the essential normality of quotient midules. Also, we will consider the relationship between the quotient modules with the Hilbert-Schmidtness of submodules. We find that all the finitely generated submodules containing a nonzero polynormial are Hilbert-Schmidt. The talk is based on the joint works with K.Guo, C.Zhao and Z.Zhu.

Brown-Halmos Characterization, and Function Theory on the Tetrablock

Paramita Pramanick
ISI Kolkata, India

We begin by establishing a Brown-Halmos type characterization of Toeplitz operators on the Hardy space associated with the tetrablock and as an application, we show that the zero operator is the only compact Toeplitz operator. Then we introduce a Schur-Agler type class associated with the tetrablock and establish a realization theorem for this class. Furthermore, we discuss a tetrablock analog of the interpolation theorem, extension theorem, and the Toeplitz corona theorem. true.

Singular Spectrum under a Wide Class of Perturbations

Constanze Liaw
University of Delaware, USA

For bounded self-adjoint operators A and K on a separable Hilbert space and a real parameter t

consider perturbed operators of the form $A + tK$. We present restrictions on the singular spectrum under trace class and more general perturbations as t is varied. To the best of our knowledge, these are the first statements on the singular spectrum under infinite rank perturbations and this work constitutes the first significant departure from the classical perturbation theory in half a century. This is joint work with Eero Saksman and Sergei Treil.

Hilbert-Schmidt norm estimates of homogeneous polynomial submodules in Hardy space over the bidisk

Chao Zu(组超)
Dalian University of Technology

In this talk, we will give a negative answer to the Hilbert-Schmidt norm boundedness problem for rank one polynomial submodules in Hardy space over the bidisk. For the homogeneous polynomial submodules $\left[(z-w)^k\right]$, by using the results about Toeplitz determinants with certain Fisher-Hartwig symbols, we show that the corresponding core operator C has eigenvalues $0, 1, \pm\frac{k}{n+k}, n \geq 1$. It implies that $2k-1 < \|C\|^2_{\text{H.S.}} < 2k+1$, which tends to infinity when k tends to infinity. Moreover, we find that there is a close connection between the Hilbert-Schmidt norm estimates for homogeneous polynomial submodules and the Fisher-Hartwig conjecture, which describes the asymptotic behavior of Toeplitz determinants for a class of functions.

The complete Nevanlinna-Pick property for sub-Hardy/Bergman space

Yixin Yang(杨义新)
Dalian University of Technology

The complete Nevanlinna-Pick property(CNP) of a reproducing kernel Hilbert space is related to the solution of Nevanlinna-Pick interpolation problems, and has been studied extensively in the literature. One of the fundamental problem is to characterize for which reproducing kernel Hilbert spaces have the CNP? We will talk about the CNP of the sub-Hardy/Bergman space on polydisk, and see that when the kernel k tensor product or Schur product with an additional factor, the CNP will diminish. This is based on the joint work with Jiming Shen.